ECONOMETRICA
JOURNAL OF THE ECONOMETRIC SOCIETY

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Source: *Econometrica*, Nov., 1986, Vol. 54, No. 6 (Nov., 1986), pp. 1425-1433

Published by: The Econometric Society

Stable URL: https://www.jstor.org/stable/1914307

Econometrica, Vol. 54, No. 6 (November, 1986), 1425–1433

A MORE POWERFUL METHOD FOR TRIANGULARIZING INPUT-OUTPUT MATRICES AND THE SIMILARITY OF PRODUCTION STRUCTURES

BY YUKIO FUKUI[1]

Studies in the production structure of any national economy, as revealed in its input-output table, have been closely related to the question of the interindustrial dependence or hierarchical structures of productive sectors leading from primary to final production. In particular, the notion of hierarchy provides a useful tool when one wants to draw inferences on the structural change or international difference of industrial structures. The standard technique for studying this notion is to triangularize the input-output table by interchanging sectors in order to maximize the entries below the main diagonal. In a perfect triangularized table, the entries above the main diagonal should be zero. For example, such a strong one-way interdependence relation as cotton-textiles-clothing can easily establish the hierarchy. Due to the existence of circular relations like coal-steel-mining equipment-coal, it is not possible to triangularize the input-output table perfectly. This paper extends and revises the previous triangulation method which is based on a permutation theorem deriving from the interchange of adjoining two industrial groups, into that among three industrial groups (Theorem 2). The new algorithm based on Theorem 2 is demonstrated by actually computing the suboptimal orderings for the four input-output tables for such more developed countries (MDC's) as the United States, Italy, Norway, Japan, and the two tables for such less developed countries (LDC's) as India and Korea. The empirical results suggest that sectors can be arranged in a similar hierarchical order among MDC's and LDC's. Transport Equipment, Machinery, Apparel, Leather and Products, Grain Mill Products, and Processed Foods, which link directly to the final demand, are recorded as higher-order sectors. Trade, Transport, and Services are lower-order ones, and Energy sectors such as Electric Power, Coal Products, Coal Mining, Petroleum, Petroleum Products and Natural Gas are the lowest. Consequently, this paper provides some more evidence in support of the similarity of hierarchical structures of production among these countries.

KEYWORDS: Hierarchy, input-output table, production structure, triangulation, interindustrial dependence, suboptimal ordering.

1. INTRODUCTION

THE NOTION OF TRIANGULARITY for the input-output table was first introduced by Chenery and Watanabe (1958), and has been commonly used as a measure for comparing the similarity of hierarchical structures of sectors from primary to final production among countries. On the computation of an approximate triangulation, Korte and Oberhofer (1970) presented a permutation theorem deriving from the interchange of adjoining two industrial groups.

This paper extends and revises their permutation theorem into that among three industrial groups (Section 2), and presents an algorithm for computing a suboptimal ordering of sectors for the interindustry transaction table (Section 3). (The proposed algorithm will be denoted by algorithm TRI.) This paper also

[1] This paper was written while the author was an instructor at Osaka Institute of Technology. I wish to thank Eizo Abe, Tetsuya Hirohata, and Katsuo Inoue for their comments on the first Japanese version and the editor and an anonymous referee for their comments and suggestions. I wish also to thank Yuzuru Ohshika and Mineo Toyokura for their encouragement. The computer program implementing this algorithm is available from the author.

revises the results by using algorithm TRI for the six countries taken from Chenery and Watanabe (1958), Santhanam and Patil (1972), and Song (1977), and provides some more evidence in support of the similarity of hierarchical structures of production among these countries (Section 4).

2. FORMULATION

Define ZN as the set of all permutations of sectors $\{1, 2, \ldots, n\}$, and let N be the identity permutation of $\{1, 2, \ldots, n\}$. For any $n \times n$ interindustry transaction table $X = (x_{ij})$ and any permutation $\pi = (\pi(1), \pi(2), \ldots, \pi(n))$, define $R(X(\pi)) = \sum_{i>j} x_{\pi(i)\pi(j)}$ $(= \sum'_{i,j \in N} x_{\pi(i)\pi(j)}$ say). The triangulation problem is stated as follows: given X, find the permutation π^* such that $R(X(\pi^*)) = \max_{\pi \in ZN} R(X(\pi))$. We call it an *absolutely optimal ordering.*[2] Since this problem belongs to the travelling salesman problem and NP-hard, it is not reasonable to find a polynomial algorithm for its exact solution. Thus, we can interpret the problem as the search for an approximate solution.

Now, let $P = (p_1, \ldots, p_s)$ and $Q = (q_1, \ldots, q_t)$ be any subsets of N with $1 \leqslant p_1 \leqslant p_s < q_1 \leqslant q_t \leqslant n$, and define λ_{PQ} as

$$(1) \qquad \lambda_{PQ} = \sum_{i \in P, j \in Q} (x_{\pi(i)\pi(j)} - x_{\pi(j)\pi(i)}).$$

Let $I = (i, \ldots, j-1)$, $J = (j, \ldots, k)$, $N2 = (I, J)$ and $N1 = (1, \ldots, i-1, k+1, \ldots, n)$ with $1 \leqslant i < j \leqslant k \leqslant l \leqslant n$. Define ZI as the set of all permutations of $s(I)(= j - i)$ elements of I, and define ZJ similarly. Let $\Gamma_{IJ} = (\pi(1), \pi(2), \ldots, \pi(J), \pi(I), \ldots, \pi(n))$ be the permutation, generated by the interchange of $\pi(I)$ and $\pi(J)$, leaving all other elements fixed. Following Korte and Oberhofer (1970) we call Γ_{IJ} *a ringshift permutation.*

Without loss of generality we take $\pi = N$. Then from (1)

$$(2) \qquad \lambda_{IJ} = \sum_{i \in I, j \in J} (x_{ij} - x_{ji}),$$

and $\Gamma_{IJ}(\pi) = (1, \ldots, J, I, \ldots, n) = \pi'$, say.

Consider the case of $s(I) \leqslant s(J)$ in the ringshift permutation. As shown in Table I, if $J2$ is taken such that $s(J2) = s(I) = j - i$, then $J1 = (j, \ldots, k - (j - i))$ and $J2 = (k - (j - i) + 1, \ldots, k)$. By the ringshift permutation,

$$(3) \qquad \pi'(\nu) = \begin{cases} \nu & \text{if } \nu \in N1, \\ \nu + j - i & \text{if } \nu \in I, J1, \\ \nu + j - k - 1 & \text{if } \nu \in J2. \end{cases}$$

THEOREM 1 (Korte and Oberhofer (1970)[3]): *If $\pi(I)$ and $\pi(J)$ in the ordering π are interchanged, then*

$$(4) \qquad R(X(\Gamma_{IJ}(\pi))) - R(X(\pi)) = \lambda_{IJ}.$$

[2] Korte and Oberhofer (1970) called it an absolutely optimal ranking (p. 48). The term "ranking" is misleading, because π is the permutation of labels of sectors (see, e.g., Santhanam and Patil (1982, p. 167)).

[3] The author has also established that Theorem 1 also holds if $s(I) > s(J)$.

TABLE I
RINGSHIFT PERMUTATION[a]

			N		
	$N1$		$N2$		$N1$
		I		J	
			$J1$	$J2$	
original ordering π	$1 \cdots$	$i \cdots j-1$	$j \cdots k-(j-i)$	$k-(j-i)+1 \cdots k$	$\cdots n$
permuted ordering π'	$1 \cdots$	$j \cdots k-(j-i)$	$k-(j-i)+1 \cdots k$	$i \cdots j-1$	$\cdots n$
		$J1$	$J2$		
			J	I	

[a] For simplicity, we take $\pi = N$.

PROOF: Without loss of generality, we take that $\pi = N$. The left-hand side of (4) can be written as

$$\sum_{i,j \in N1}' (x_{\pi'(i)\pi'(j)} - x_{ij}) + \sum_{i \in N1, j \in N2}' (x_{\pi'(i)\pi'(j)} - x_{ij})$$
$$+ \sum_{i \in N2, j \in N1}' (x_{\pi'(i)\pi'(j)} - x_{ij}) + \sum_{i,j \in N2}' (x_{\pi'(i)\pi'(j)} - x_{ij}).$$

While the first, second, and third terms vanish from (3), for the fourth term we observe

$$\begin{aligned} \sum_{i,j \in N2}' (x_{\pi'(i)\pi'(j)} - x_{ij}) &= \sum_{i,j \in I,J1}' (x_{\pi'(i)\pi'(j)} - x_{ij}) + \sum_{i,j \in J2}' (x_{\pi'(i)\pi'(j)} - x_{ij}) \\ &\quad + \sum_{i \in J2, j \in I,J1} (x_{\pi'(i)\pi'(j)} - x_{ij}) \\ &= \sum_{i,j \in J}' x_{ij} - \sum_{i,j \in I,J1}' x_{ij} + \sum_{i,j \in I}' x_{ij} - \sum_{i,j \in J2}' x_{ij} \\ &\quad + \sum_{i \in I, j \in J} x_{ij} - \sum_{i \in J2, j \in I,J1} x_{ij} \\ &= \sum_{i \in I, j \in J} x_{ij} - \sum_{i \in J, j \in I} x_{ij}. \end{aligned}$$

Hence, from (2), we get (4). *Q.E.D.*

Here, let $I = (i, \ldots, j-1)$, $J = (j, \ldots, k)$, and $K = (k+1, \ldots, l)$ with $1 \leqslant i < j \leqslant k \leqslant l \leqslant n$ be the adjoining subsets of N and define another permutation $\Gamma_{IJK}(\pi) = (\pi(1), \ldots, \pi(K), \pi(J), \pi(I), \ldots)$, generated by the interchange of $\pi(I)$ and $\pi(K)$, leaving all other elements fixed. Also define λ_{IJK} as

$$\lambda_{IJK} = \lambda_{IJ} + \lambda_{JK} + \lambda_{IK}. \tag{5}$$

THEOREM 2: *If $\pi(I)$ and $\pi(K)$ are interchanged in the ordering π, then*

$$R(X(\Gamma_{IJK}(\pi))) - R(X(\pi)) = \lambda_{IJK}.$$

PROOF: Without loss of generality we assume that $\pi = N$. Then $\Gamma_{IJK}(N) = (1, \ldots, L, I, \ldots n)$ with $L = (K, J)$. From the definitions

$$\Gamma_{JK}(\pi) = (1, \ldots, I, L, \ldots) \quad \text{and} \quad \Gamma_{IL} \cdot \Gamma_{JK}(\pi) = \Gamma_{IJK}(\pi).$$

Thus, using

$$\begin{aligned} R(X(\Gamma_{IL} \cdot \Gamma_{JK}(\pi))) - R(X(\Gamma_{JK}(\pi))) &= \sum_{i \in I, j \in L} (x_{ij} - x_{ji}) \\ &= \sum_{i \in I, j \in K} (x_{ij} - x_{ji}) \\ &\quad + \sum_{i \in I, j \in J} (x_{ij} - x_{ji}) \\ &= \lambda_{IK} + \lambda_{IJ}, \end{aligned}$$

then,

$$\begin{aligned} R(X(\Gamma_{IL} \cdot \Gamma_{JK}(\pi))) - R(X(\pi)) &= R(X(\Gamma_{IL} \cdot \Gamma_{JK}(\pi))) \\ &\quad - R(X(\Gamma_{JK}(\pi))) + R(X(\Gamma_{JK}(\pi))) \\ &\quad - R(X(\pi)) \\ &= \lambda_{IK} + \lambda_{IJ} + \lambda_{JK} \\ &= \lambda_{IJK}, \end{aligned}$$

the last step following from (5). *Q.E.D.*

Before proceeding, it is helpful to compare Theorem 1 with Theorem 2. The latter generalizes the former by interchanging not only adjoining subsets of N but also disjoining subsets. The following two corollaries are special cases of Theorem 2.

COROLLARY 1 (Helmstadter (1964)): *If sector i and sectors from $i+1$ to k are interchanged, then $R(X(\pi))$ is increased by $\sum'^{k}_{l=i+1} (x_{il} - x_{li})$.*

PROOF: Follows from Theorem 2 by taking $I = (i)$ and $J = (i+1, \ldots, k)$. *Q.E.D.*

COROLLARY 2 (Tovissi, Spircu, and Tasnadi (1978)): *If sector i and sector l are interchanged, $R(X(\pi))$ is increased by*

$$\sum_{j=i+1}^{l-1} (x_{jl} - x_{lj}) + (x_{il} - x_{li}) + \sum_{j=i+1}^{l-1} (x_{ij} - x_{ji}).$$

PROOF: Follows from Theorem 2 by taking $I = (i)$, $J = (i+1, \ldots, l-1)$, and $K = (l)$. *Q.E.D.*

THEOREM 3: *If $\pi = \pi^*$, then*

$$\lambda_{IJK} = R(X(\Gamma_{IJK}(\pi^*))) - R(X(\pi^*)) \leq 0 \quad \textit{for all } I, J \textit{ and } K.$$

PROOF: If there exists $\pi = \pi^{**}$ such that

$$R(X(\pi^*)) < R(X(\Gamma_{IJK}(\pi^*))) = R(X(\pi^{**})),$$

then it is contradictory to the assumption of π^*. *Q.E.D.*

The theorem contains the following corollary as a special case.

COROLLARY 3 (Belkin (1981)): $\lambda_{IJ} \leqslant 0$ *for all I and J, if* $\pi = \pi^*$.

3. ALGORITHM TRI

We introduce the following definitions.

DEFINITION 1 (Korte and Oberhofer (1970, p. 486)): *A relatively optimal ordering of the first type* (π_1) is any ordering such that $R(X(\Gamma_{IJ}(\pi_1))) \leqslant R(X(\pi_1))$ for all I and J.

DEFINITION 2: *A relatively optimal ordering of the second type* (π_2) is any ordering such that $R(X(\Gamma_{IJK}(\pi_2))) \leqslant R(X(\pi_2))$ for all I, J, and K.

DEFINITION 3: *A relatively optimal ordering of the third type* (π_3) is any ordering such that $R(X(\Gamma_{ik}(\pi_3))) \leqslant R(X(\pi_3))$ for all i and k ($i \neq k$).

Our technique for detecting π_2 is founded on the enumeration approach. The process begins with $\pi = \pi^{(1)}$. As a short-cut method to choose the initial solution, we utilize the Simpson–Tsukui ordering (1965) by setting $\pi^{(1)} = (\pi_m, \pi_n, \pi_e, \pi_s)$ where the subscripts m, n, e, and s denote metal, nonmetal, energy, and service blocks respectively, and arranging sectors within each block in the decreasing order of the ratio of purchased inputs to total product. At each augmentation of $R(X(\pi))$, $\pi^{(k+1)}$ is constructed from $\pi^{(k)}$ as follows: (i) Choose the maximum $\lambda_{IJK}^{(k+1)} = R(X(\Gamma_{IJK}^{(k+1)}(\pi^{(k)}))) - R(X(\pi^{(k)}))$ for all I, J, and K; (ii) interchange I and K satisfying (i); (iii) set $\Gamma_{IJK}^{(k+1)}(\pi^{(k)}) = \pi^{(k+1)}$. The algorithm repeats all steps until $\lambda_{IJK}^{(k+1)}$ becomes zero or negative. It should be noted that Algorithm TRI requires more repetitive calculation than the Korte–Oberhofer (KO) procedure, because there are ${}_{n+1}C_3$ ways to choose I and J for Γ_{IJ} (Korte and Oberhofer (1970, pp. 485–486)) and ${}_{n+2}C_4$ ways to choose I, J, and K for Γ_{IJK}. This can be shown in the following way. (a) There are ${}_nC_4$ ways for the choice of distinct i, j, k, and l of $(1, 2, \ldots, n)$ when $K \neq \varnothing$ and J contains more than two elements; (b) ${}_nC_3$ ways for $i, j (= k)$, and l when $K \neq \varnothing$ and J has a single element; (c) ${}_nC_3$ ways for i, j, and k when $K = \varnothing$ and J contains more than two elements; (d) ${}_nC_2$ for i and $j (= k)$ when $K = \varnothing$ and J has a single element. Thus ${}_nC_3 + {}_nC_2 = {}_{n+1}C_3$ (from (c) and (d)) is less than ${}_nC_4 + {}_nC_3 + {}_nC_3 + {}_nC_2 = {}_{n+2}C_4$.

THEOREM 4: *If we take the same ordering of sectors as a starting solution for Algorithm TRI, then*

$$R(X(\pi_1)) \leqslant R(X(\pi_2)) \leqslant R(X(\pi^*))$$

and

$$R(X(\pi_3)) \leqslant R(X(\pi_2)) \leqslant R(X(\pi^*)).$$

PROOF: The searching regions induced by Algorithm TRI (from (a) through (d)) include those by the KO procedure (from (c) and (d)). *Q.E.D.*

COROLLARY 4: *If $\pi = \pi_1$ or π_3, then*

$$x_{\pi(i+1)\pi(i)} \geqslant x_{\pi(i)\pi(i+1)} \qquad \text{for any } i.$$

The proof is straightforward and can be omitted.

REMARK: Simpson and Tsukui (1965, pp. 444–446) proposed a handy method of triangulation. Taking $I=(i)$, $J=(i+1,\dots,l-1)$, and $K=(l)$, they set as a starting point $i=1$ and $l=i+1$, and increase only l one by one until $\lambda_{IJ}+\lambda_{IK}>0$. Then halt and move I down behind K, and continue this step. Otherwise they apply this process for $i=i+1$. Their method does not guarantee finding π_2, because $\lambda_{IJ}+\lambda_{IK}>0$ does not generally mean $\lambda_{IJ}+\lambda_{IK}+\lambda_{JK}>0$.

4. AN EXAMPLE

At first, we introduce the following three conventional measures of the hierarchical structure of production: (i) the hierarchical order of sectors itself; (ii) degree of linearity $\lambda(\pi)$, defined by

$$\lambda(\pi) = R(X(\pi))/T,$$

where $T=\sum_{i\neq j}^{n} x_{ij}$ (Lamel, Richter, and Teufelsbauer (1971, p. 62)); (iii) Spearman rank correlation (r_s) between different rankings for international or intertemporal comparisons (Chenery and Watanabe (1958, p. 496)).

Algorithm TRI has been used to compute the relatively optimal ordering of the second type for the six input–output tables: four standardized 29-sector tables for such more developed countries (MDC's) as the United States in 1947, Japan in 1951, Italy in 1950, and Norway in 1950 and two tables for such less developed countries (LDC's) as Korea in 1970 (29-sector) and India in 1964–65 (22-sector).

Three different rankings for the MDC's have been known as suboptimum: (i) Chenery and Watanabe's country ranking; (ii) their compromise ranking; (iii) a revised version of (i) and (ii) given by Helmstadter (1964). Santhanam and Patil (1972) presented their orderings for India. Using these orderings as starting solutions, we first compute the degrees of linearity. Then we check whether these orderings are π_2's and if not, we repeat the algorithm until π_2 is attained.[4]

[4] If $n=29$, there are 31465, 4060, and 406 ways for Γ_{IJK}, Γ_{IJ}, and Γ_{ik} respectively. Notice that $n! = 8.84176\times 10^{30}$.

Table II presents the estimated $\lambda(\pi)$ of the starting solution (S), that of the final solution (F), and the number of iterations (I). As can be seen in the table, Helmstadter's orderings (iii) generally approximate the π_2's well. In particular his ordering of Italy proves to be π_2. For India we calculate S and F to be .8096 and .8901. As Song does not present his ranking for the Korean table, we take π_2's for the above MDC's shown in Table II as starting solutions to attain Korea's unique ordering. The estimated $\lambda(\pi_2)$ is .8614.

Table III suggests that sectors can be arranged in a similar order among MDC's and Korea. Transport Equipment, Machinery, Apparel, Leather and Products, Grain Mill Products, and Processed Foods, which link directly to final demand, are recorded as higher-order sectors. Trade, Transport, and Services are lower-order ones, and Energy sectors such as Electric Power, Coal Products, Coal Mining, Petroleum, Petroleum Products and Natural Gas are the lowest. Notice that due to the existence of symmetric submatrices in diagonal blocks of X,[5] *in fact in almost all diagonal, multiple solutions* (π's) with the same value of λ can be found: for example forty solutions for Japan, eighty for Italy, three hundred and sixty for Norway, and two for Korea. Only in the United States, can a unique ordering be identified.[6]

To compare the ranking of sectors among pairs of countries in the triangular arrangement statistically, r_s's are computed as shown in Table IV. The following three points are clear from the table. (i) Chenery and Watanabe overestimated r_s; (ii) Song underestimated r_s; and (iii) Santhanam and Patil underestimated r_s. Point (i) does not, however, deny the similarity of production structures of the MDC's. Rather we confirm that production structures of these countries are triangular in form, since r_s's are still high and hierarchical ordering is similar among the MDC's. Further from (ii) and (iii) the assertion of the hierarchical similarity holds also between LDC's and MDC's. Finally (ii) suggests that our

TABLE II
ESTIMATED DEGREE OF LINEARITY[a]

	USA			Japan			Italy			Norway		
	S	(I)	[F]	S	(I)	[F]	S	(I)	[F]	S	(I)	[F]
(i)	.7981	7	.8333[b]	.8284	6	.8340[b]	.9360	5	.9380[b]	.8311	6	.8382
(ii)	.7951	10	.8333[b]	.8167	10	.8336	.9118	11	.9380[b]	.7230	15	.8409[b]
(iii)	.8332	1	.8333[b]	.8338	2	.8340[b]	.9380[b]	0	.9380[b]	.8379	1	.8409[b]

[a] S denotes the starting solution, (I) denotes the number of iterations, and [F] denotes the final solution.
[b] Means the best final solution of each country.

[5] For example, if $x_{\pi(i)\pi(i+1)} = x_{\pi(i+1)\pi(i)}$, then there are such two sequences of sectors as $\pi(1), \ldots, \pi(i), \pi(i+1), \ldots$, and $\pi(1), \ldots, \pi(i+1), \pi(i), \ldots$, with the same value of the degree of linearity.

[6] Using the same label as Santhanam and Patil's (1972) for sectors, the ordering of sectors for India (π_2) is as follows: 2, 6, 7, 9, 14, 1, 3, 11, 4, 8, 20, 15, 13, 10, 17, 18, 12, 21, 19, 5, 16, 22. Not only the ordering of the top five sectors (except the following one-way dependency: $2 \leftarrow 6$, $6 \leftarrow 7$, and $6 \leftarrow 9$) but also that of sector 11 and 4, and that of 18 and 12 are indifferent. Thus $25 \times 2 \times 2 = 100$ ways of π_2's can be identified.

TABLE III
Ordering of Sectors in Triangular Arrangement[a]

	Label of Sector	Ranking	U.S.A.	Japan	Italy	Norway	Korea
1	Apparel	[1]	9	4	2 9	4	9
2	Shipbuilding	[2]	1	5	12 12	6	1
3	Leather and Products	[3]	3	2	1	1	3
4	Processed Foods	[4]	4	1	17	2	4
5	Fishing	[5]	6	9		9	5
6	Grain Mill Products	[6]	5	17	3	3	6
7	Transport	[7]	2	12	4	12	8
8	Industry n.e.c.	[8]	12	8	5	14	10
9	Transport Equipment	[9]	8	13	8	10	11
10	Rubber Products	[10]	15	14	10	8	18
11	Textiles	[11]	10	10	13	25	12
12	Machinery	[12]	11	11	14	15	15
13	Iron and Steel	[13]	16	6	25	11	14
14	Nonmetalic Mineral Products	[14]	18	3	15	18	17
15	Lumber and Wood Products	[15]	13	15	11	16	13
16	Chemicals	[16]	21	18	6	5	21
17	Printing and Publishing	[17]	14	16	18	7	22
18	Agriculture and Forestry	[18]	19	21	16	24	16
19	Nonmetallic Minerals	[19]	22	23	21	28	25
20	Petroleum Products	[20]	24	22	19	17	23
21	Nonferrous Metals	[21]	7	24	22	13	26
22	Metal Mining	[22]	28	25	23	21	27
23	Coal Products	[23]	17	19	7	19	7
24	Trade	[24]	25	7	24	22	2
25	Paper and Products	[25]	26	26	28	23	28
26	Electric Power	[26]	23	27	26	26	20
27	Coal Mining	[27]	27	28	27	20	19
28	Services	[28]	20	20	20	27	24
29	Petroleum and Natural Gas	[29]	29	29	29		29

[a] The ordering of sectors included in an open brace is indifferent, whereas those included in a closed bracket must be arranged in that order.

TABLE IV
Spearman Rank Correlations Among Pairs of Countries[a]

	Japan	Italy	Norway	Korea	India[b]
U.S.A.	.817(.945)	.801(.902)	.857(.901)	.817[.504]	.901{.646}
Japan		.930(.868)	.755(.863)	.785[.708]	.684{.598}
Italy			.814(.905)	.733[.482]	.921{.712}
Norway				.690[.635]	.890{.696}

[a] Numbers in parentheses are estimates by Chenery and Watanabe (1958), numbers in brackets are those by Song (1977), and numbers in braces are those by Santhanam and Patil (1972).
[b] We compile the five input-output tables at the same level of aggregation as the Indian table.

conclusion is different from Song's (1977, p. 160) assertion, "production structures are ... more similar in within the same country than in international comparison between different countries."[7]

5. SUMMARY AND A SUGGESTION FOR FURTHER RESEARCH

We provide some more evidence in support of the similarity of hierarchical structures of production across the six countries. Surprisingly enough, our results suggest that the similarity can be supported more strongly than the previous studies of Santhanam and Patil (1972), and Song (1977).

A suggestion for further work could be made. It will be interesting to examine the hierarchy of sectors across each country, using international input-output tables in the context of international interdependence among countries.

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Manuscript received January 1985; final revision received January, 1986.

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[7] Song (1977) reported intertemporal r_s's as follows: .941 for 1970 and 1966, .435 for 1970 and 1963, and .438 for 1970 and 1960. As he does not present the data except for 1970, we cannot reestimate these figures.